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COMBINATION OF RAYLEIGH AND DYNAMIC EDGE EFFECT METHODS IN STUDYING VIBRATIONS OF RECTANGULAR PLATES

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The dynamic edge effect method (DEEM) suggested by Bolotin has been used extensively in solving problems of natural vibrations for elastic rectangular plates and also for structures consisting of them [1]. Generally speaking the method intended for finding high natural frequencies and forms under kinetic boundary conditions also gives good results for low forms of vibration [2]. With existence of static conditions at the contour the accuracy of determining low natural values decreases [3]. The error of the DEEM is connected with the fact that the solution constructed by means of it does not satisfy the original problem in the vicinity of boundaries. One possible way for refining the method is construction of angular boundary layers [4], and another is combination of the asymptotic method with variation methods. Combination of the DEEM with the Rayleigh–Ritz method was the subject of [5], although there only the case of kinematic boundary conditions was considered, and therefore it is difficult to check the efficiency of this approach. Equations obtained in [5] for natural frequencies are only applicable for a square plate clamped along all edges. Of particular interest with this combination is estimation of the first approximation (Rayleigh equation) since in this case it is possible to obtain an expression for natural frequency in closed form.

In the present work an asymptotic expression is obtained by combining Rayleigh and DEEM methods for the frequency of natural vibrations suitable for arbitrary unchanged conditions at the boundary along the rectilinear edge, and the efficiency of this approach has been studied.

We consider vibration of an elastic rectangular ($0 \le x_1 \le a_1$, $0 \le x_2 \le a_2$) plate. According to Rayleigh the expression for frequency parameter λ has the form

$$\lambda = a_1 a_2 \left[\left(\rho h/D\right) \int_0^{a_1}\int_0^{a_2} \left(w_{,11}^2 + w_{,22}^2 + 2\nu w_{,11} w_{,22} + 2(1-\nu) w_{,12}^2\right) dx_1 dx_2 \Bigg/ \left(\int_0^{a_1}\int_0^{a_2} w^2 dx_1 dx_2\right)\right]^{1/2}. \tag{1}$$

Here $\lambda = \omega a_1 a_2 (\rho h/D)^{1/2}$; w is normal deflection; ν is Poisson's ratio; ω is natural frequency; h is thickness; ρ is material density; $D = Eh^3/[12(1-\nu^2)]$; E is Young's modulus.

The expression for the function of deflection obtained by means of the DEEM [1] is written as

$$f(x_1, x_2) = S_1(x_1)\sin(\beta_1 x_2 + l_2) + S_2(x_2)\sin(\beta_1 x_1 + l_1); \tag{2}$$

$$S_i(x_i) = \sin(\beta_i x_i + l_i) + C_{i1}\exp(\alpha_i x_i) + C_{12}\exp(-\alpha_i x_i)\ (i = 1, 2). \tag{3}$$

We take the expression for deflection $w(x_1, x_2)$ in the form

$$w(x_1, x_2) = S_1(x_1) S_2(x_2). \tag{4}$$

Dnepropetrovsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 6, pp. 164-166, November-December, 1988. Original article submitted October 12, 1987.

TABLE 1

λ by RBM	Diverg-ence from SM, %	λ by SM [6]	λ by DEEM	Diverg-ence from SM,%	λ by RBM	Diverg-ence from SM, %	λ by SM [6]	λ by DEEM	Diverg-ence from SM,%
13,97	3,6	13,47	11,31	19,1	64,71	1,6	63,69	60,93	4,5
22,21	3,5	21,98	22,21	3,5	74,46	1,3	73,51	71,86	2,3
35,95	3,2	34,81	33,01	5,4	106,99	1,4	105,31	103,31	2,1

From (1), (3), (4) an equation follows for frequency parameter valid for arbitrary conditions at the edges:

$$\lambda = a_1a_2\{(\rho h/D)[K_1 + K_2 - 2\nu K_3 + 2(1-\nu)K_4]/K_0\}^{1/2}. \tag{5}$$

Here $K_0 = (\mathbf{A}_1\xi)\cdot(\mathbf{A}_2\xi)$; $K_1 = (\mathbf{A}_1\eta_1)(\mathbf{A}_2\xi)$; $K_2 = (\mathbf{A}_1\xi)(\mathbf{A}_2\eta_2)$; $K_3 = (\mathbf{A}_1\varkappa_1)(\mathbf{A}_2\varkappa_2)$; $K_4 = (\mathbf{B}_1\theta_1)(\mathbf{B}_2\theta_2)$. Vector components are as follows:

$$\xi = \{1;\ 2;\ 1\},\quad \eta_j = \{\beta_j^4;\ -2\alpha_j^2\beta_j^2;\ \alpha_j^4\},$$
$$\varkappa_j = \{-\beta_j^2;\ \alpha_j^2-\beta_j^2;\ \alpha_j^2\},\quad \theta_j = \{\beta_j^2;\ 2\alpha_j\beta_j;\ \alpha_j^2\},$$
$$\mathbf{A}_j = \{A_{1j};\ A_{2j};\ A_{3j}\},\ \mathbf{B}_j = \{A_{4j};\ A_{5j};\ A_{6j}\}\ (j = 1, 2).$$

Coordinates A_{ij} are found by the equations

$$A_{1j} = \{z/2 - [\sin 2(\beta_j z + l_j)]/(4\beta_j)\}\big|_0^{a_j},$$
$$A_{2j} = \{(\alpha_j^2+\beta_j^2)^{-1}[\alpha_j F_{4j}F_{1j} - \beta_j F_{3j}F_{2j}]\}\big|_0^{a_j},$$
$$A_{3j} = \{F_{5j}/(2\alpha_j) + 2C_{j1}C_{j2}z\}\big|_0^{a_j},$$
$$A_{4j} = \{z/2 + [\sin 2(\beta_j z + l_j)]/(4\beta_j)\}\big|_0^{a_j},$$
$$A_{5j} = \{(\alpha_j^2+\beta_j^2)^{-1}[\alpha_j F_{3j}F_{2j} + \beta_j F_{4j}F_{1j}]\}\big|_0^{a_j},$$
$$A_{6j} = \{F_{5j}/(2\alpha_j) - 2C_{j1}C_{j2}z\}\big|_0^{a_j}\quad (j = 1, 2),$$

where $F_{1j} = \sin(\beta_j z + \ell_j)$; $F_{2j} = \cos(\beta_j z + \ell_j)$; $F_{3j} = C_{j1}\exp(\alpha_j z) + C_{j2}\exp(-\alpha_j z)$; $F_{4j} = C_{j1}\exp(\alpha_j z) - C_{j2}\exp(-\alpha_j z)$; $F_{5j} = C_{j1}^2\exp(2\alpha_j z) - C_{j2}^2\exp(-2\alpha_j z)$ $(j = 1, 2)$.

We apply the algorithm given above to finding natural frequencies for a square plate with a contour free from forces. The set of transcendental equations for finding unknown wave numbers in this case has the form [4] $\beta_j a_j = 2L_j + m_j\pi$ with $L_j = \arctan\{(\beta_j/\alpha_j)[\beta_j^2 + (2-\nu)\beta_k^2]^2/(\beta_j^2 + \nu\beta_k^2)^2\}$ $(j = 1, 2,\ k = 1, 2,\ j \neq k,\ m_j = 0, 1, 2, \ldots)$. Constants $\alpha_j = (\beta_j^2 + 2\beta_k^2)^{1/2}$ $(j = 1, 2,\ k = 1, 2,\ j \neq k)$. In terms of wave numbers from boundary conditions constants ℓ_i and C_{ij} $(i, j = 1, 2)$ in Eq. (3), and consequently also functions $S_1(x_1)$, $S_2(x_2)$ [1] are determined.

Results of calculating dimensionless frequency λ by the Rayleigh—Bolotin method (RBM) are given above, by the series method (SM) [6], and by the traditional DEEM are given in Table 1 (with $\nu = 0.3$). It is necessary to note that in the solution obtained by the SM [6] six terms are retained in the series, and therefore the result for calculating the basic tone apparently exhibits high accuracy.

Comparison shows that the present method makes it possible to refine considerably the results found by the Bolotin method for the first frequency. Use of the RBM and DEEM, giving the upper and lower estimates for natural values, respectively, for the highest forms makes it possible to obtain quite narrow boundaries for the interval in which natural frequency is found. With an increase in the number of forms both solutions approach the accurate solution asymptotically.

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RESULTS OF DETERMINING LIMITING DYNAMIC COMPRESSION DIAGRAMS FOR SANDY SOILS AND CLAY

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UDC 624.131+539.215

A method was described in [1, 2] for determining limiting dynamic compression diagrams corresponding to instantaneous loading ($\dot{\varepsilon} = \infty$) for soils and porous materials sensitive to deformation rate. The method is based on the relationship of weak-disturbance propagation rates with the limiting dynamic diagram $\varphi(\varepsilon)$ with compression of a viscoplastic material. However, actual data for determining $\varphi(\varepsilon)$ in [1, 2] was only obtained for air-dried sandy soil. Given below are the results of experimental studies for determining these diagrams for sandy soils with different moisture contents, and also for dense clays.

It is assumed in the same way as in [1, 2] that the main properties of sandy and clay soils under short-term dynamic loads with sufficient accuracy are described with uniaxial compression (under plane strain conditions) by a deformation rule

$$\frac{\partial \varepsilon}{\partial t} = \frac{1}{E(\sigma_1, \varepsilon)} \frac{\partial \sigma_1}{\partial t} + g(\sigma_1 - f(\varepsilon)), \quad E(\sigma_1, \varepsilon) = \begin{cases} E(\varepsilon), \dfrac{\partial \sigma_1}{\partial t} \geqslant 0, \\ E_*(\sigma_1, \varepsilon), \dfrac{\partial \sigma_1}{\partial t} < 0, \end{cases} \qquad (1)$$

where σ_1 is the greatest principal stress; $E(\varepsilon)$, $E_*(\sigma_1, \varepsilon)$ are functions determined by experiment with loading ($\partial\sigma_1/\partial t \geq 0$) and with unloading ($\partial\sigma_1/\partial t < 0$); $g(z) > 0$ with $z = \sigma_1 - f(\varepsilon) > 0$ and $g(z) \equiv 0$ with $z \leq 0$; $f(\varepsilon)$ is the static compression diagram for the material with $\dot{\varepsilon} \to \infty$.

As shown in [2], for a material of type (1) the relationship for a small-disturbance propagation rate $c(\varepsilon)$ and limiting dynamic diagram $\varphi(\varepsilon)$ ($\dot{\varepsilon} = \infty$) under load is determined by the relationship

$$E(\varepsilon) = \frac{d\varphi(\varepsilon)}{d\varepsilon} = \rho_0 c^2(\varepsilon) \qquad (2)$$

(ρ_0 is initial material density). By integrating, from (2) we obtain the limiting dynamic diagram

$$\varphi(\varepsilon) = \int_0^\varepsilon E(\xi)\, d\xi, \quad \dot{\varepsilon} = \infty. \qquad (3)$$

Thus, by knowing from an experiment the relationship $c(\varepsilon)$ it is possible to plot the limiting dynamic diagram $\varphi(\varepsilon)$ ($\dot{\varepsilon} = \infty$) with loading.

Testing was carried out in a UDN-150 unit [1, 2] fitted with a system for measuring weak-disturbance propagation rates in a compressed material. As in [1], compression was created as

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 6, pp. 166-169, November-December, 1988. Original article submitted October 6, 1987.

